LESSONS - ADVANCED MATH

Polynomials and Rewriting Expressions

$$x^3 + 6x^2 + 7x - 6 = (x^2 + bx - 2)(x + 3)$$

1. In the given equation, b is a constant. If the equation is true for all values of x, what is the value of b?

   A) 2
   B) 3
   C) 7
   D) 9

2. Which of the following is equivalent to $16s^4 - 4t^2$?

   A) $4(s^2 - t)(4s^2 + t)$
   B) $4(4s^2 - t)(s^2 + t)$
   C) $4(2s^2 - t)(2s^2 + t)$
   D) $(8s^2 - 2t)(8s^2 + 2t)$

3. Which expression is equivalent to $xy^2 + 2xy^2 + 3xy$?

   A) $2xy^2 + 3xy$
   B) $3xy^2 + 3xy$
   C) $6xy^4$
   D) $6xy^5$

Quadratic Functions and Equations

4. A car is traveling at x feet per second. The driver sees a red light ahead, and after 1.5 seconds reaction time, the driver applies the break. After the brake is applied, the car takes $\frac{x}{24}$ seconds to stop, during which time the average speed of the car is $\frac{x}{2}$ feet per second. If the car travels 165 feet from the time the driver saw the red light to the time it comes to a complete stop, which of the following equations can be used to find the value of x?

A) $x^2 + 48x - 3,960 = 0$
B) $x^2 + 48x - 7,920 = 0$
C) $x^2 + 72x - 3,960 = 0$
D) $x^2 + 72x - 7,920 = 0$

5. What are the solutions to the equation $x^2 - 3 = x$?

A) $\frac{-1\pm\sqrt{11}}{2}$
B) $\frac{-1\pm\sqrt{13}}{2}$
C) $\frac{1\pm\sqrt{11}}{2}$
D) $\frac{1\pm\sqrt{13}}{2}$

6. If $x > 0$ and $2x^2 + 3x - 2 = 0$, what is the value of x?

7. What is the sum of the solutions to $(2x - 1)^2 = (x + 2)^2$?

Exponential Functions, Equations, and Expressions and Radicals

8. A researcher estimates that the population of a city is increasing at an annual rate of 0.6%. If the current population of the city is 80,000, which of the following expressions appropriately models the population of the city t years from now according to the researcher's estimate?

   A) $80,000(1 + 0.006)^t$
   B) $80,000(1 + 0.006^t)$
   C) $80,000 + 1.006^t$
   D) $80,000(0.006^t)$

9. Which of the following is equivalent to $\left(\frac{1}{\sqrt{x}}\right)^n$, where $x > 0$?

   A) $x^{\frac{n}{2}}$

   B) $x^{-\frac{n}{2}}$

   C) $x^{n+\frac{n}{2}}$

   D) $x^{n-\frac{n}{2}}$

$$x - 12 = \sqrt{x + 44}$$

10. What are all possible solutions to the given equation?

    A) 5

    B) 20

    C) −5 and 20

    D) 5 and 20

Solving Rational Equations

$$\frac{3}{t+1} = \frac{2}{t+3} + \frac{1}{4}$$

11. If t is a solution to the given equation and $t > 0$, what is the value of t?

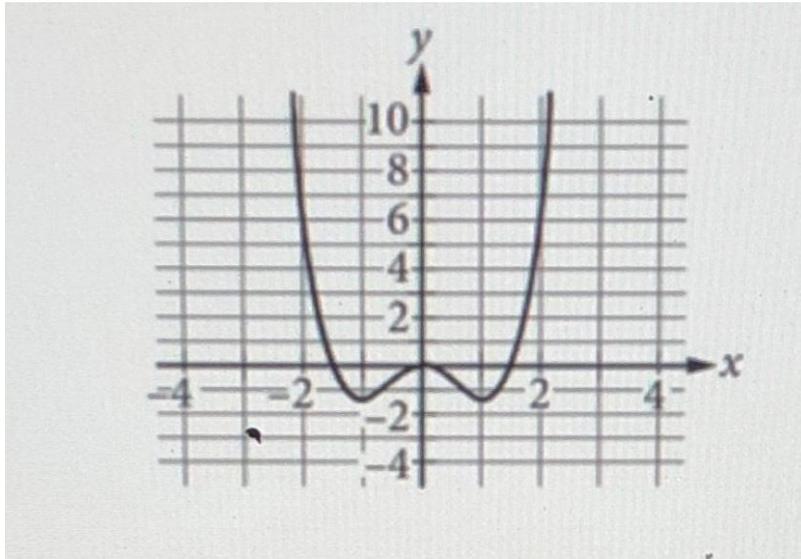**Systems of Equations**

$$3x + y = -3$$

$$(x + 1)^2 - 4(x + 1) - 6 = y$$

12. If (x,y) is a solution to the given system of equations and $y > 0$, what is the value of y?

Relationships between Algebraic and Graphical Representations of Functions

13. The graph of which of the following functions in the xy-plane has x-intercepts at -4 and 5?

   (A) $f(x) = (x + 4)(x - 5)$

   (B) $f(x) = (x - 4)(x + 5)$

   (C) $f(x) = (x - 4)^2 + 5$

   (D) $f(x) = (x + 5)^2 - 4$

14. The function $f(x) = x^4 - 2.4x^2$ is graphed in the xy-plane where $y = f(x)$, as shown. If k is a constant such that the equation $f(x) = k$ has 4 solutions, which of the following could be the value of k?

   (A) 1
   (B) 0
   (C) -1
   (D) -2

Function Notation

15. If $g(x) = 2x + 1$ and $f(x) = g(x) + 4$, what is $f(2)$?

Interpreting and Analyzing More Complex Equations in Context

16. For a certain reservoir, the function f gives the water level $f(n)$, to the nearest whole percent of capacity, on the nth day of 2016. Which of the following is the best interpretation of $f(37) = 70$?

    (A) The water level of the reservoir was at 37% capacity for 70 days in 2016.
    (B) The water level of the reservoir was at 70% capacity for 37 days in 2016.
    (C) On the 37th day of 2016, the water level of the reservoir was at 70% capacity.
    (D) On the 70th day of 2016, the water level of the reservoir was at 37% capacity.

17. If an object of mass m is moving at speed v, the object's kinetic energy is (KE) is given by the equation $KE = \frac{1}{2}mv^2$. If the mass of the object is halved and its speed is doubled, how does the kinetic energy change?

    (A) The kinetic energy is halved.
    (B) The kinetic energy is unchanged.
    (C) The kinetic energy is doubled.
    (D) The kinetic energy is quadrupled (multiplied by a factor of 4).

18. A gas in a container will escape through holes of microscopic size, as long as the holes are larger than the gas molecules. This process is called effusion. If a gas of molar mass M_1 effuses at a rate of r_1 and a gas of molar mass M_2 effuses at a rate of r_2, then the following relationship holds.

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

This is known as Graham's law. Which of the following correctly expresses M_2 in terms of M_1, r_1 and r_2 ?

(A) $M_2 = M_1 \left(\frac{r_1^2}{r_2^2}\right)$

(B) $M_2 = M_1 \left(\frac{r_2^2}{r_1^2}\right)$

(C) $M_2 = \sqrt{M_1} \left(\frac{r_1}{r_2}\right)$

(D) $M_2 = \sqrt{M_1} \left(\frac{r_2}{r_1}\right)$

19. A store manager estimates that if a video game is sold at a price of p dollars, the store will have weekly revenue, in dollars, of $r(p) = -4p^2 + 200p$ from the sale of the video game. Which of the following equivalent forms of $r(p)$ shows, as constants or coefficients, the maximum possible weekly revenue and the price that results in the maximum revenue?

(A) $r(p) = 200p - 4p^2$
(B) $r(p) = -2(2p^2 - 100p)$
(C) $r(p) = -4(p^2 - 50p)$
(D) $r(p) = -4(p - 25)^2 + 2{,}500$

1. B	11. 5
2. C	12. 6
3. B	13. A
4. D	14. C
5. D	15. 9
6. $\dfrac{1}{2}$	16. C
7. $\dfrac{8}{3}$	17. C
8. A	18. A
9. B	19. D
10. B	

Answer explanations can be found in the *Digital SAT Study Guide* on pages 229 to 242.

Summary: Advanced Math

Main Purpose: To assess your understanding of nonlinear relationships.

Portion of the Test: About 35 percent (13 to 15 questions)

Question Types:

- Equivalent expressions
- Nonlinear equations in one variable and systems of equations in two variables
- Nonlinear functions